Permutations and Combinations

Module-I

Collection
(set / types)

Choose only two

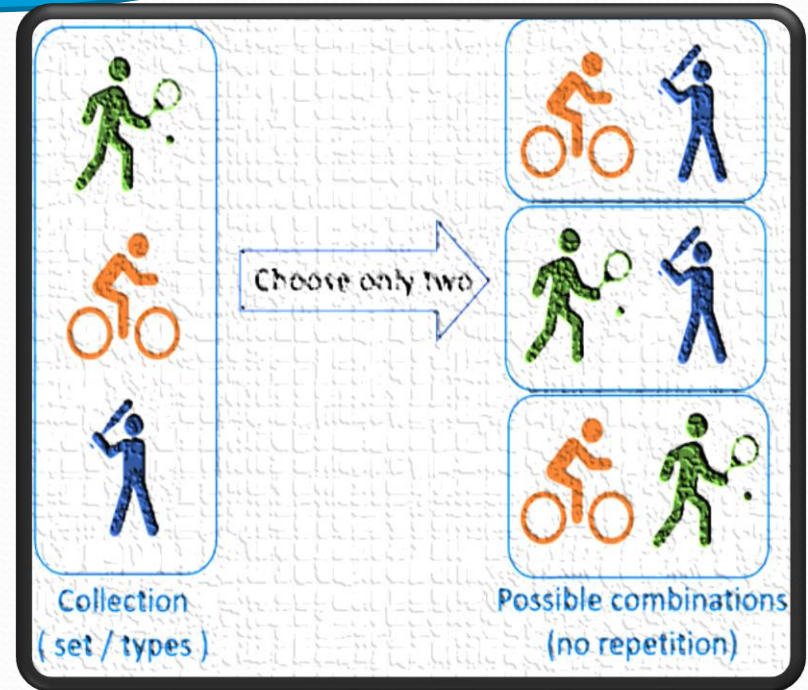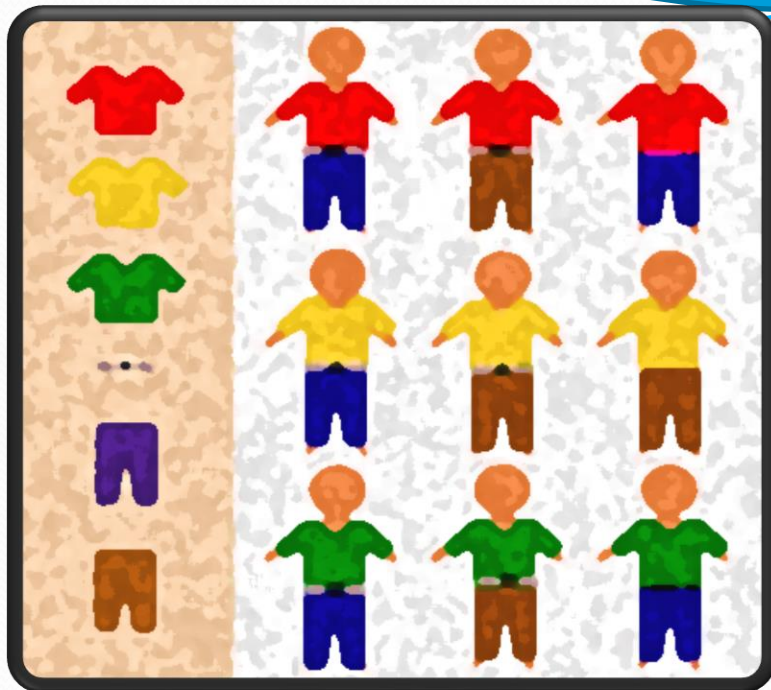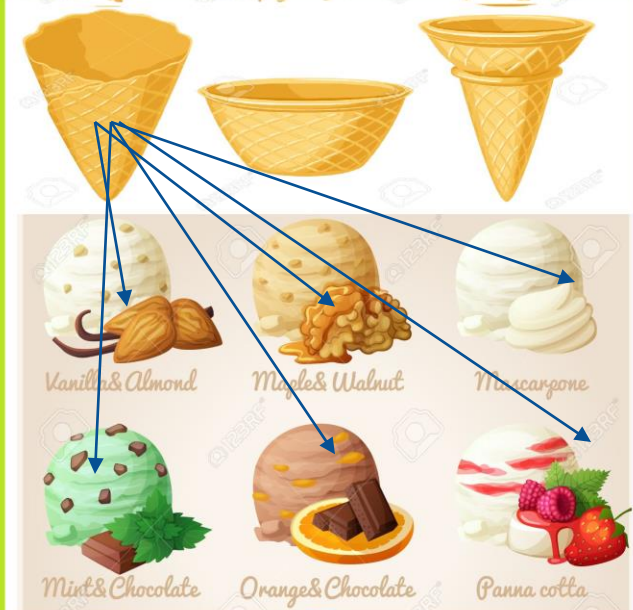Possible combinations
(no repetition)

Can you answer quickly? ??

1. You have 3 shirts and 4 pants .How many total number of outfits you can have ?
2. There are 6 flavors of ice-cream and 3 different cones. How many choices of ice-cream you can have?
3. A restaurant offers 3 starters and 6 main courses, how many different choices does a diner have when ordering?

How many outfits?

Alex now has four different shirts and three different pants.

To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Yogurt Parfait (choose 1 of each)		
Flavor	**Fruit**	**Nuts**
Plain	Peaches	Almonds
Vanilla	Strawberries	Peanuts
	Bananas	Walnuts
	Raspberries	
	Blueberries	

A farmer wants to buy a tractor and a trailer. He has three choices for a tractor and three for a trailer. How many different sets of equipment has he to choose from?

History

Permutations and **combinations**, the various ways in which objects from a set may be selected, generally without replacement, to form subsets. This selection of subsets is called a permutation when the order of selection is a factor, a combination when order is not a factor. By considering the ratio of the number of desired subsets to the number of all possible subsets for many games of chance in the 17th century, the French mathematicians Blaise Pascal and Pierre de Fermat gave impetus to the development of combinatorics and probability theory.

Fundamental Counting Principle

Fundamental Counting Principle can be used to determine the number of possible outcomes when there are two or more characteristics.

Fundamental Counting Principle states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m * n$ possible outcomes for the two events together.
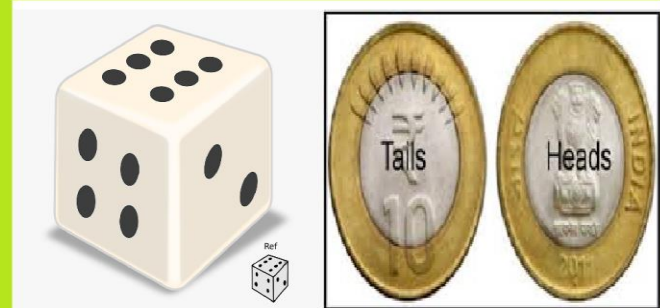
Fundamental Counting Principle

Lets start with a simple example.

A student is to roll a die and flip a coin.
How many possible outcomes will there be?

1H	2H	3H	4H	5H	6H
1T	2T	3T	4T	5T	6T

6*2 = 12 outcomes

12 outcomes

Ref

Tails   Heads

Example: You are buying a new car

There are **2** body styles:

sedan or hatchback
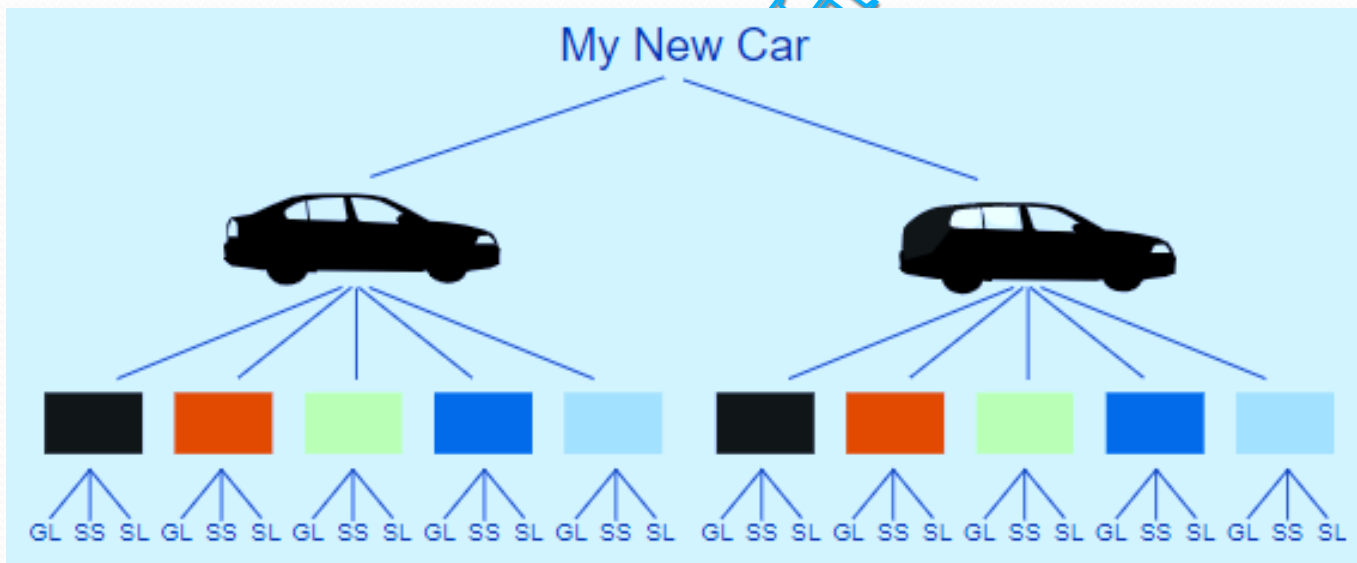
There are **5** colors available:
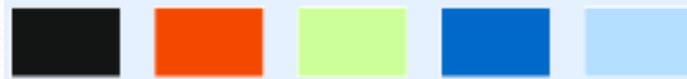
There are **3** models:

- GL (standard model),
- SS (sports model with bigger engine)
- SL (luxury model with leather seats)

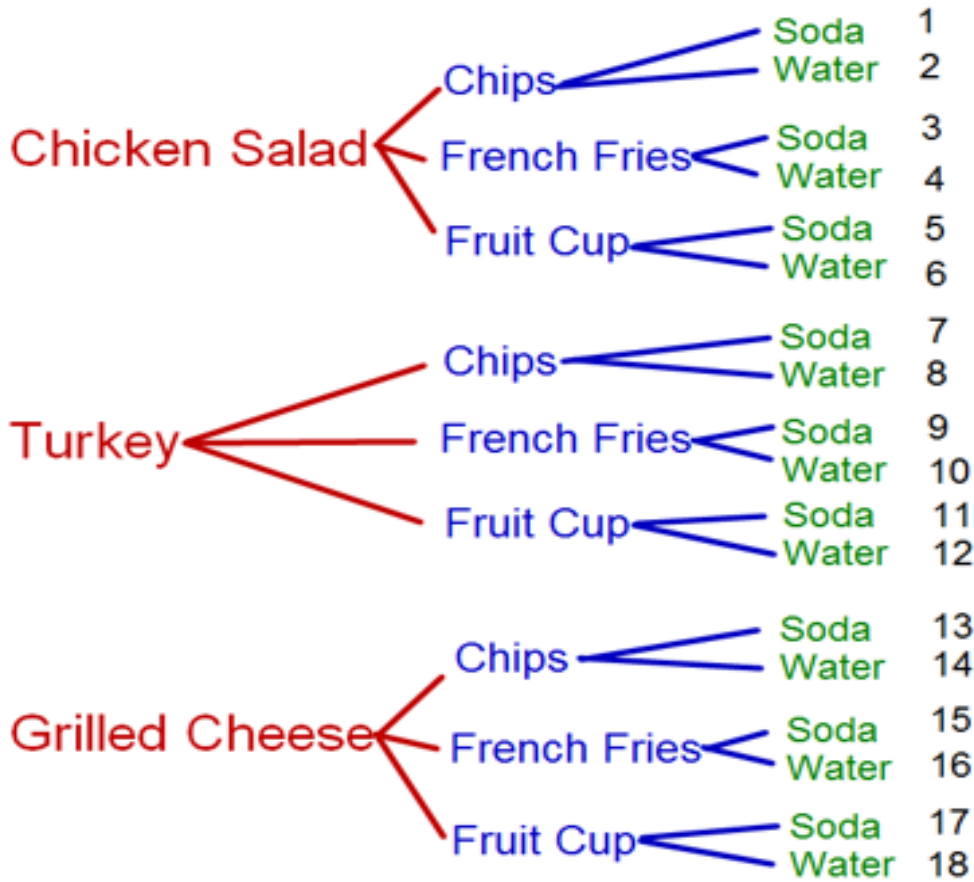How many total choices?

You can see in this "tree" diagram:

My New Car

GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL

You can count the choices, or just do the simple calculation:

Total Choices 2x5x3 =30

Chicken Salad
- Chips
  - Soda — 1
  - Water — 2
- French Fries
  - Soda — 3
  - Water — 4
- Fruit Cup
  - Soda — 5
  - Water — 6

Turkey
- Chips
  - Soda — 7
  - Water — 8
- French Fries
  - Soda — 9
  - Water — 10
- Fruit Cup
  - Soda — 11
  - Water — 12

Grilled Cheese
- Chips
  - Soda — 13
  - Water — 14
- French Fries
  - Soda — 15
  - Water — 16
- Fruit Cup
  - Soda — 17
  - Water — 18

Visiting a Turkish restaurant

$3 \times 3 \times 2 = 18$

There are 18 total combinations

Fundamental Principle of Counting

Basic Counting Principles

Example

How many ways can you answer a
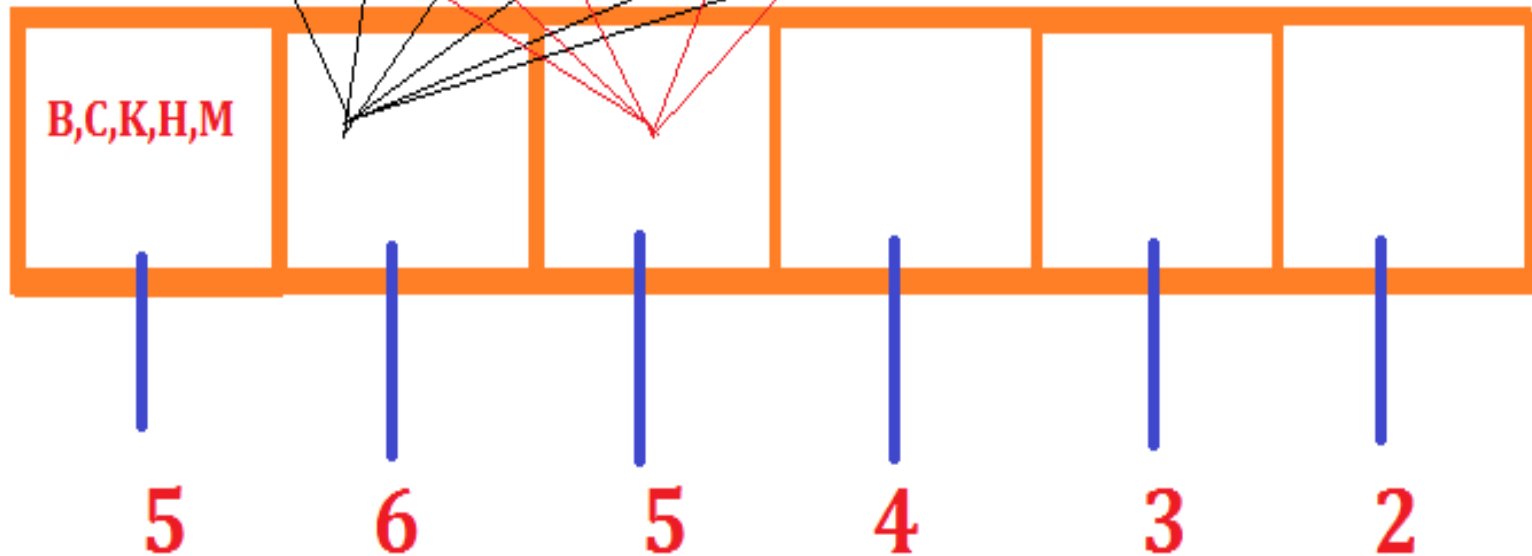
a. 20-item true or false quiz?

   20 x 2 = 40

b. 20-item multiple choice test, with choices A, B, C, D?

   20 x 4 = 80

Find the number of ways to form 6 letter word from the letters B, E, C, K, H, A, M with the condition that it must start with a consonant. **REPETITION NOT ALLOWED**

TOTAL= 7 LETTERS

B C K H M   A E

B,C,K,H,M					
5	6	5	4	3	2

5x 6x5x4x3x2 = 3600

Let's discuss:

Question 1:

How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

(i) repetition of the digits is allowed?

(ii) repetition of the digits is not allowed?

Answer 1:

(i) There will be as many ways as there are ways of filling 3 vacant places in succession by the given five digits. In this case, repetition of digits is allowed.
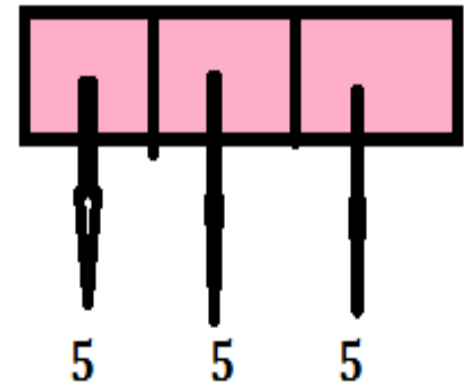
Therefore, the units place can be filled in by any of the given five digits.

Similarly, tens and hundreds digits can be filled in by any of the given five digits.

Thus, by the multiplication principle, the number of ways in which three-digit numbers can be formed from the given digits is $5 \times 5 \times 5 = 125$

$1, 2, 3, 4, 5$

5	5	5

$5 \times 5 \times 5 = 125$

## (ii) repetition of the digits is not allowed?

In this case, repetition of digits is not allowed. Here, if units place is filled in first, then it can be filled by any of the given five digits.

Therefore, the number of ways of filling the units place of the three-digit number is 5.

Then, the tens place can be filled with any of the remaining four digits and the hundreds place can be filled with any of the remaining three digits.

Thus, by the multiplication principle, the number of ways in which three-digit numbers can be formed without repeating the given digits is $5 \times 4 \times 3 = 60$

1 , 2 , 3 , 4 , 5

5   4   3
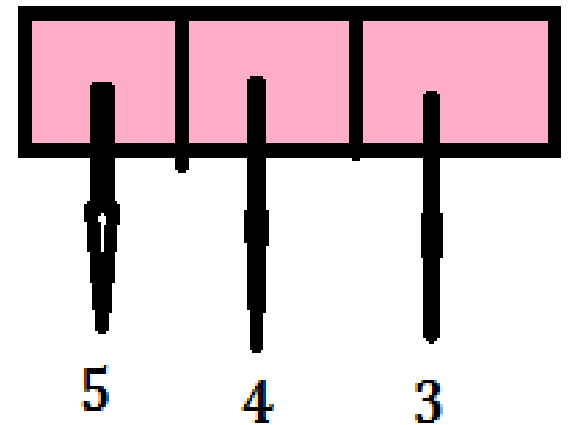
5x4x3 = 60

Question 2:

How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Answer

There will be as many ways as there are ways of filling 3 vacant places in succession by the given six digits.

In this case, the units place can be filled by 2 or 4 or 6 only i.e., the units place can be filled in 3 ways.

The tens place can be filled by any of the 6 digits in 6 different ways and also the hundreds place can be filled by any of the 6 digits in 6 different ways, as the digits can be repeated.

Therefore, by multiplication principle, the required number of three digit even numbers is

$3 \times 6 \times 6 = 108$

1 , 2 , 3 , 4 , 5 ,6

2,4,6

6   6
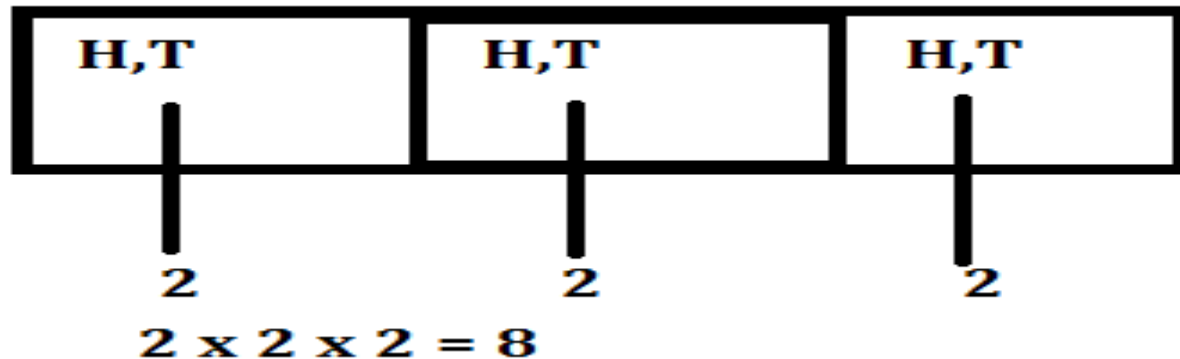
3
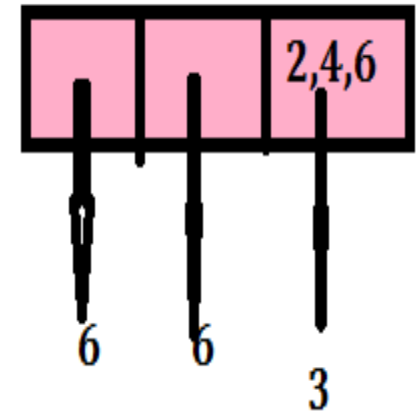
$6 \times 6 \times 3 = 108$

Question: 3

A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Answer

When a coin is tossed once, the number of outcomes is 2 (Head and tail) i.e., in each throw, the number of ways of showing a different face is 2.

Thus, by multiplication principle, the required number of possible outcomes is 2 × 2 × 2 = 8

Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Answer Each signal requires the use of 2 flags.

There will be as many flags as there are ways of filling in 2 vacant places in succession by the given 5 flags of different colours.

The upper vacant place can be filled in 5 different ways by any one of the 5 flags following which, the lower vacant place can be filled in 4 different ways by any one of the remaining 4 different flags.

Thus, by multiplication principle, the number of different signals that can be generated is $5 \times 4 = 20$.

5

5 x 4 = 20

4
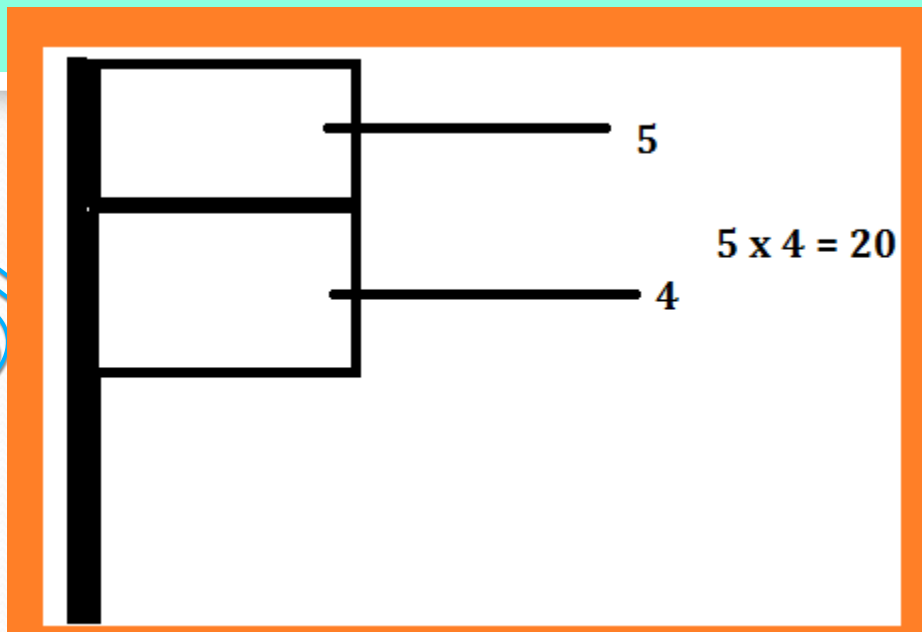
Try these.....

A password for a site consists of 4 digits followed by 2 letters. The letters A and Z are not used, and each digit or letter many be used more than once. How many unique passwords are possible?

A test consists of 15 multiple choice questions, with each question having four possible answers. In how many different ways can a student check off one answer to each question?

A "make-your-own-adventure" story lets you choose 6 starting points, gives 4 plot choices, and then has 5 possible endings. How many adventures are there?

1. In a cricket match, how many choices can be made for man of the match ?
2. In a class there are 22 girls and 17 boys. The teacher wants to select either a girl or a boy to represent the class in a function. In how many ways can the teacher make this selection?
3. In a class there are 17 girls and 22 boys. In how many ways can the teacher form a team of one girl and one boy from amongst the students of the class to represent the school in a quiz competition?
4. Of 11 cricket players, one is to be chosen as captain and another as vice captain. How many choices are there?
5. Lata wants to go abroad by air and return by ship. She has a choice of 6 different airlines to go and 4 different ships to return. In how many ways she can perform her journey ?
6. There are 4 doors in Lotus temple. In how many ways can a person enter the temple and leave by a different door ?
7. Four persons A, B, C and D are to give lectures to an audience. In how many ways can the organiser arrange the order of their presentation?
8. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE (repetition of letters not allowed). (NCERT)
9. For a group photograph, 3 boys and 2 girls stand in a line in all possible ways. How many photos could be taken if each photo corresponds to each such arrangement?
10. Six pictures are to be arranged (in line from left to right) on a wall of an art gallery for display. How many arrangements are possible ?

THANK YOU

INDIAN SCHOOL MUSCAT

Stay safe

Stay blessed